FORMULÁRIO (26-3-2019)

α – alfa
β – beta
χ – qui
δ – delta
ε – eps
λ – lambda
μ – miu
ν – niu
π – pi
θ – theta
ρ – rho
τ – tau
ω – omega
Ω – OMEGA
ξ – qsi

$l = s_o, \quad l' = s_i$

$n = n_1, \; n' = n_2$

Refracção

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

$$\theta_{Crítico} = \mathrm{asin}(n_2 / n_1)$$

$$\theta_{Brewster} = \mathrm{atan}(n_2 / n_1)$$

Prismas

$$\delta \approx (n-1)\alpha$$

$$\delta = \theta_{i1} - \alpha + \mathrm{asin}\left[\sqrt{n^2 - \sin^2(\theta_{i1})}\sin(\alpha) - \sin(\theta_{i1})\cos(\alpha)\right]$$

$$\theta_{i1} = (\delta_m + \alpha)/2$$

$$n = \sin\left(\frac{\delta_m + \alpha}{2}\right) \Big/ \sin\left(\frac{\alpha}{2}\right)$$

Equações de Fresnel

$$r_{//} = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_i \cos\theta_t + n_t \cos\theta_i} \qquad t_{//} = \frac{2 n_i \cos\theta_i}{n_i \cos\theta_t + n_t \cos\theta_i}$$

$$r_{\perp} \equiv \left(\frac{E_{0r}}{E_{0i}}\right) = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t} \qquad t_{\perp} \equiv \left(\frac{E_{0t}}{E_{0i}}\right) = \frac{2 n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t}$$

$$R_{\perp} = r_{\perp}^2 \qquad R_{\parallel} = r_{\parallel}^2$$

$$T_{\perp} = \left(\frac{n_t \cos\theta_t}{n_i \cos\theta_i}\right) t_{\perp}^2 \qquad T_{\parallel} = \left(\frac{n_t \cos\theta_t}{n_i \cos\theta_i}\right) t_{\parallel}^2$$

$$\mathrm{T} = \frac{4 n_1 n_2}{(n_1 + n_2)^2} \qquad \Re = \left(\frac{n_2 - n_1}{n_1 + n_2}\right)^2$$

$\tan \theta_B = n_t / n_i$

$\sin \theta_c = n_t / n_i$

Óptica Geométrica

$$\frac{n_1}{s_o} + \frac{n_2}{s_i} = K \qquad m \equiv M_T \equiv \frac{h'}{h} = -\frac{n\, l'}{n'\, l}$$

$$K \equiv \frac{n}{f} = \frac{n'}{f'} \qquad \frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

$$M_L = -{M_T}^2$$

$$K = K_1 + K_2 - \frac{d}{n_l} K_1 K_2$$

$$\delta = -\frac{n'}{n_1'}\frac{d K_1}{K}$$

$$\delta' = -\frac{n}{n_1'}\frac{d K_2}{K} = \delta \frac{n\, K_2}{n'\, K_1}$$

$$K_{L-espessa} = (n_l - n_{meio})\left(\frac{1}{R_1} - \frac{1}{R_2} + \frac{d(n_l - n_{meio})}{n_l R_1 R_2}\right)$$

$$\delta = -\frac{f(n_l - 1)d}{n_l R_2}$$

$$\delta' = -\frac{f(n_l - 1)d}{n_l R_1} = \delta \frac{R_2}{R_1}$$

$$K = -\frac{2}{R} \qquad K = \frac{n_2 - n_1}{R}$$

$$\nabla^2 u(\mathbf{r}, \mathrm{t}) - \frac{1}{c^2}\frac{\partial^2 u(\mathbf{r}, t)}{\partial t^2} = 0$$

$$(\nabla^2 + k^2) U(\mathbf{r}) = 0$$

$$u\,(\mathbf{r}, t) = U(\mathbf{r}) e^{-i 2\pi \nu t} = U(\mathbf{r}) e^{-i\omega t}$$

$$k = \frac{2\pi\nu}{c} = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

$$U(\mathbf{r}) = A e^{-i\mathbf{k}.\mathbf{r}}$$

$$U(\mathbf{r}) = \frac{A}{r} e^{-ikr}$$

$$U(\mathbf{r}) = \frac{A}{z} e^{-ikz} e^{-ik\frac{x^2+y^2}{2z}}$$

Ondas paraxiais

$$U(\mathbf{r}) = A(\mathbf{r}) e^{-ikz}$$

$$\nabla_T^2 A - i2k\frac{\partial \mathrm{A}}{\partial \mathrm{z}} = 0$$

Modo Gaussiano TEM_{00}

$$U(\rho, z) = A_0 \frac{W_0}{W(z)} e^{-\frac{\rho^2}{W^2(z)}} e^{-i\left[kz + \frac{k\rho^2}{2R(z)} - \zeta(z)\right]} \qquad z_0 = \frac{\pi {W_0}^2}{\lambda}$$

$$W(z) = W_0\left[1 + \left(\frac{z}{z_0}\right)^2\right]^{1/2} \qquad R(z) = z\left[1 + \left(\frac{z_0}{z}\right)^2\right] \qquad \zeta(z) = \mathrm{atan}(z / z_0)$$

Modos de Hermite / Laguerre - Gauss

$$U_{l,m}(x, y, z) = A_{l,m}\frac{W_0}{W(z)}\mathbb{G}_l\left[\frac{\sqrt{2}x}{W(z)}\right]\mathbb{G}_m\left[\frac{\sqrt{2}y}{W(z)}\right] e^{\left[-ikz - ik\frac{x^2+y^2}{2R(z)} + (l+m+1)\zeta(z)\right]} \qquad \mathbb{G}_l(u) = \mathbb{H}_l(u) e^{-\frac{u^2}{2}}$$

$$U_{l,m}(\rho, \varphi, z) = A_{l,m}\frac{W_0}{W(z)}\left(\frac{\rho}{W(z)}\right)^{|l|} L_m^{|l|}\left[\frac{2\rho^2}{W^2(z)}\right] e^{-\frac{\rho^2}{W^2(z)}} e^{\left[-ikz - ik\frac{\rho^2}{2R(z)} - il\varphi + (l+2m+1)\zeta(z)\right]} \qquad \rho^2 = x^2 + y^2$$

Modos de Bessel

$$U(\mathbf{r}) = A(x, y) e^{-i\beta z}$$

$$\nabla_T^2 A + k_T^2 A = 0$$

$$k_T^2 + \beta^2 = k^2$$

$$\nabla_T^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$$

$$A(x, y) = A_m J_m(k_T \rho) e^{-im\phi}, \qquad m = 0, \pm 1, \pm 2, \ldots$$

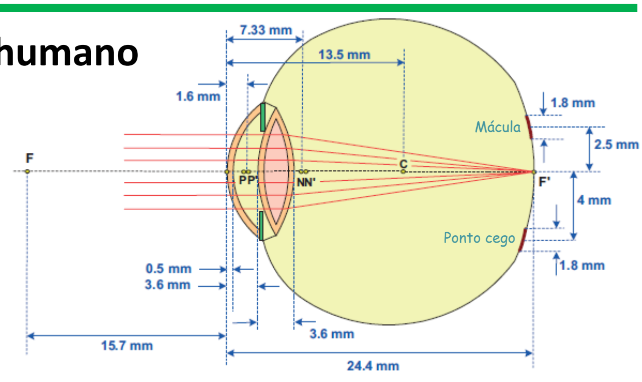
	NÃO ACOMODADO (NA)				ACOMODADO (A)			
SUPERFÍCIE	Z	R	n	K	Z	R	n	K
Córnea - 1ª superfície	0	7,7000	1,3760	48,83	0,0000	7,7000	1,3760	48,8300
Córnea - 2ª superfície	0,5	6,8000	1,3360	-5,88	0,5000	6,8000	1,3360	-5,8800
Cristalino - 1ª superfície	3,6	10,0000	1,4085	5	3,2000	5,3300	1,4260	9,3750
Cristalino - 2ª superfície	7,2	-6,0000	1,3360	8,33	7,2000	-5,3300	1,3360	9,3750

Posições relativas ao vértice da córnea (z=0), em mm

		CÓRNEA		CRISTALINO		OLHO COMPLETO		
GRANDEZA FÍSICA		NA	A	NA	A	NA	A	
Potência	K	43,053	43,053	19,11	33,06	58,636	70,57	K
Ponto Principal Objecto	Z(H)	-0,0496	-0,0496	5,678	5,145	1,348	1,772	Z(H)
Ponto Principal Imagem	Z(H')	-0,0506	-0,0506	5,807	5,225	1,602	2,086	Z(H')
Ponto Focal Objecto	Z(F)					-15,707	-12,397	Z(F)
Ponto Focal Imagem	Z(F')					24,387	21,016	Z(F')
Distância focal Objecto	f	23,227	23,227	69,908	40,416	17,055	14,619	f
Distância focal Imagem	f'	31,031	31,031	69,908	40,416	22,785	18,93	f'
Ponto Nodal Objecto	Z(N)					7,078	6,533	Z(N)
Ponto Nodal Imagem	Z(N')					7,332	6,847	Z(N')
Posição da Pupila de Entrada						3,045	2,667	
Posição da Pupila de Saída						3,664	3,211	

Distâncias focais e posições relativas ao vértice da córnea (z=0), em mm

Olho humano



Difracção

$$U(x_0,y_0) = \frac{1}{i\lambda} \iint_{\Sigma} U(x_1,y_1)\frac{e^{ikr_{01}}}{r_{01}} cos\theta \, dx_1 dy_1$$

$$U(x,y) = \frac{e^{ikz}}{i\lambda z}\iint_{-\infty}^{\infty} U(\xi,\eta)e^{-i\frac{k}{2z}\left[(x-\xi)^2+(y-\eta)^2\right]}d\xi d\eta$$

$$U(x,y) = \frac{e^{ikz}e^{-i\frac{k}{2z}(x^2+y^2)}}{i\lambda z}\iint_{-\infty}^{\infty} U(\xi,\eta)e^{-i\frac{2\pi}{\lambda z}(x\xi+y\eta)}d\xi d\eta = \frac{e^{ikz}e^{-i\frac{k}{2z}(x^2+y^2)}}{i\lambda z}\mathcal{F}\{U\}_{f_X=\frac{x}{\lambda z},\, f_Y=\frac{y}{\lambda z}}$$

Transformação de Fourier

$$\mathcal{F}\{g\} = G(f_X,f_Y) = \iint_{-\infty}^{\infty} g(x,y)e^{-i2\pi(f_X x+f_Y y)}dxdy$$

$$\iint_{-\infty}^{\infty} |g(x,y)|^2 dxdy = \iint_{-\infty}^{\infty} |G(f_X,f_Y)|^2 df_X df_Y$$

$$\mathcal{F}^{-1}\{G\} = g(x,y) = \iint_{-\infty}^{\infty} G(f_X,f_Y)\, e^{+i2\pi(f_X x+f_Y y)}df_X df_Y$$

$$g \star\star\, \delta = \iint_{-\infty}^{\infty} g(x,y)\,\delta(x-\xi,y-\eta)\,dxdy = g(\xi,\eta)$$

$$\mathcal{F}\left\{\iint_{-\infty}^{\infty} g(\xi,\eta)\,h(x-\xi,y-\eta)\,d\xi d\eta\right\} = G(f_X,f_Y)H(f_X,f_Y)$$

$$\mathcal{F}\mathcal{F}^{-1}\{g(x,y\} = \mathcal{F}^{-1}\mathcal{F}\{g(x,y\} = g(x,y)$$

$$\mathcal{F}\{gh\} = G(f_X,f_Y) \star\star\, H(f_X,f_Y)$$

$$\mathcal{F}\{\alpha g+\beta h\} = \alpha\mathcal{F}(g)+\beta\mathcal{F}(h) = \alpha G(f_X,f_Y)+\beta H(f_X,f_Y)$$

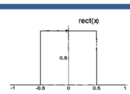
$$\mathcal{F}\{g(x-a,y-b)\} = G(f_X,f_Y)\,e^{-i2\pi(f_X a+f_Y b)}$$

$$\mathcal{F}\left\{\iint_{-\infty}^{\infty} g(\xi,\eta)\,g^*(\xi-x,\eta-y)\,d\xi d\eta\right\} = \mathcal{F}\{g\otimes\otimes g\} = |G(f_X,f_Y)|^2$$

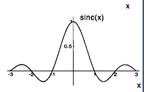
$$\mathcal{F}\{g(ax,by)\} = \frac{1}{|ab|}G\left(\frac{f_X}{a},\frac{f_Y}{b}\right)$$

J. Goodman, Introduction to Fourier Optics (3ª edição, 2005), Cap. 2

$$\text{rect}(x) = \begin{cases} 1 & |x| < \frac{1}{2} \\ \frac{1}{2} & |x| = \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$

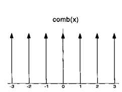


$$\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$



$$\text{circ}(\sqrt{x^2+y^2}) = \begin{cases} 1 & \sqrt{x^2+y^2} < 1 \\ \frac{1}{2} & \sqrt{x^2+y^2} = 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\text{comb}(x) = \sum_{n=-\infty}^{\infty}\delta(x-n)$$



$$\text{comb}\frac{x-x_0}{b} = |b|\sum_{n=-\infty}^{\infty}\delta(x-x_0-nb)$$

Function	Transform
$\exp[-\pi(a^2x^2+b^2y^2)]$	$\frac{1}{\|ab\|}\exp\left[-\pi\left(\frac{f_X^2}{a^2}+\frac{f_Y^2}{b^2}\right)\right]$
$\text{rect}(ax)\,\text{rect}(by)$	$\frac{1}{\|ab\|}\text{sinc}(f_X/a)\,\text{sinc}(f_Y/b)$
$\delta(ax,by)$	$\frac{1}{\|ab\|}$
$\exp[j\pi(ax+by)]$	$\delta(f_X-a/2,\, f_Y-b/2)$
$\text{comb}(ax)\,\text{comb}(by)$	$\frac{1}{\|ab\|}\text{comb}(f_X/a)\,\text{comb}(f_Y/b)$
$\exp[j\pi(a^2x^2+b^2y^2)]$	$\frac{j}{\|ab\|}\exp\left[-j\pi\left(\frac{f_X^2}{a^2}+\frac{f_Y^2}{b^2}\right)\right]$
$\exp[-(a\|x\|+b\|y\|)]$	$\frac{1}{\|ab\|}\frac{2}{1+(2\pi f_X/a)^2}\frac{2}{1+(2\pi f_Y/b)^2}$

$$\cos(2\pi f_0 x) = \frac{e^{+i2\pi f_0 x}+e^{-i2\pi f_0 x}}{2}$$

$$\frac{1}{2}[\delta(f_X-f_0)+\delta(f_X+f_0)]$$

$$\sin(2\pi f_0 x) = \frac{e^{+i2\pi f_0 x}-e^{-i2\pi f_0 x}}{2i}$$

$$\frac{1}{2i}[\delta(f_X-f_0)-\delta(f_X+f_0)]$$

Dieléctricos. Metais

$$\mathcal{P} = \epsilon_o\chi\mathcal{E}+\mathcal{P}_{\text{NL}},$$

$$\mathcal{P}_{\text{NL}} = 2\text{d}\mathcal{E}^2+4\chi^{(3)}\mathcal{E}^3+\cdots$$

$$k = \beta - i\frac{\alpha}{2} = k_0\sqrt{1+\chi'+i\chi''}$$

$$n - j\frac{1}{2}\frac{\alpha}{k_o} = \sqrt{\epsilon/\epsilon_o} = \sqrt{1+\chi'+j\chi''}$$

$$\chi(\nu) = \chi_0\frac{\nu_0^2}{\nu_0^2-\nu^2+j\nu\,\Delta\nu}$$

$$\chi'(\nu) = \chi_0\frac{\nu_0^2\,(\nu_0^2-\nu^2)}{(\nu_0^2-\nu^2)^2+(\nu\,\Delta\nu)^2}$$

$$\chi''(\nu) = -\chi_0\frac{\nu_0^2\,\nu\,\Delta\nu}{(\nu_0^2-\nu^2)^2+(\nu\,\Delta\nu)^2}$$

Absorção fraca

$$n \approx \sqrt{1+\chi'}$$

$$\alpha \approx -\frac{k_o}{n}\chi''.$$

$$\epsilon_e = \epsilon + \frac{\sigma}{j\omega}$$

$$n \approx \sqrt{\sigma/2\omega\epsilon_o}$$

$$\alpha \approx \sqrt{2\omega\mu_o\sigma}$$

$$\eta \approx (1+j)\sqrt{\omega\mu_o/2\sigma}$$

$$d_p = 1/\alpha = 1/\sqrt{2\omega\mu_o\sigma}$$

$$\varepsilon_e = \varepsilon_0\left(1-\frac{\omega_p^2}{\omega^2}\right)$$

$$k = \beta - j\tfrac{1}{2}\alpha = \omega\sqrt{\epsilon_e\mu_o}$$

$$\omega_p = \sqrt{\frac{\sigma}{\varepsilon_0\tau_c}} = \sqrt{\frac{Ne^2}{m\varepsilon_0}}$$

$$n - j\alpha/2k_o = \sqrt{\epsilon_e/\epsilon_o}$$